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A High-Accuracy Gyroscope Test of General Relativity and the Search for a Massless Scalar Field

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A high-accuracy orbiting relativity-gyroscope experiment is described. A pure unsupported gyroscope in a spinning drag-free satellite at ambient temperatures with conventional optical instrumentation can measure the relativity drifts with errors as low as 3 to 0.1 μas/yr, an improvement of 10^2 to 10^3 over the current GP-B experiment. Recent theoretical work has suggested that the PPN parameter $1-\gamma \approx 1/\omega_{BD}$ might lie in the range 10^{-4} to 10^{-8} signaling the presence of a massless scalar field. The experiment described here could measure γ to an accuracy between 10^{-7} and 10^{-8}; but for the highest accuracies, the measurement depends critically on either future microarcsecond astrometry or on a two-gyro version of the experiment.

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Schouten [1], Fokker [2], Pugh [3], and Schiff [4] have suggested using a gyroscope to test general relativity. In the *parameterized post-Newtonian* (PPN) formulation of metric theories of gravity, the average drift with respect to the distant stars of a perfect gyro in an orbit around a spherical earth with no other astronomical bodies present is

$$\dot{\boldsymbol{\phi}}=\left(\gamma+\frac{1}{2}\right)\frac{(M_\oplus G)^{3/2}\hat{\mathbf{h}}}{c^2 r_s^{5/2}}+\left(\frac{\gamma+1}{2}+\frac{\alpha_1}{8}\right)\frac{(\mathbf{J}_\oplus-3\hat{\mathbf{h}}(\hat{\mathbf{h}}\cdot\mathbf{J}_\oplus))G}{2c^2 r_s^3}. \quad (1)$$

$\hat{\mathbf{h}}$ is the unit orbital angular momentum, $\mathbf{J}_\oplus$ is the earth's angular momentum, r_s is the distance to the satellite from the center of the earth, and γ and α_1 are two PPN parameters [5]. The two terms in Eq. (1) are known as the geodetic and frame-dragging drifts and have values in a polar orbit at 750 km of about 6400 and 39 mas/yr. In general relativity $\gamma = 1$ and $\alpha_1 = 0$. In the Jordan-Brans-Dicke and scalar-tensor theories with ω_{BD} as the coupling parameter, $\gamma = (\omega_{BD}+1)/(\omega_{BD}+2)$ and $\alpha_1 = 0$. Nominally $\dot{\phi}$ is measured relative to a star in the plane of a polar orbit.

Recently interest in scalar-tensor theories of gravity has been revived by the ubiquitous presence of scalar fields in extensions of the standard model and by the realization that a scalar-tensor theory could exit from inflation in a non-fine-tuned way. In addition, two papers in the last year have explained why $1-\gamma \approx 1/\omega_{BD}$ might be very small: these theories contain a mechanism which makes them tend to general relativity as the universe ages [6, 7]. The resulting value of $1-\gamma$ has been calculated, and it is expected to lie between 10^{-4} and 10^{-8}.

For a uniquely interpretable scientific result, it is important that the experiment measure the altitude dependence as well as the value of γ since an altitude variation different from Eq. (1) would indicate a non-PPN theory of gravity. Conversely, a non-PPN theory could mimic the existence of a scalar field if the altitude dependence were not measured and if the experiment accuracy were insufficient to give a concordant measurement of γ from both the geodetic and frame-dragging drifts. For example, at least two non-PPN theories of gravity, the theory of a light massive scalar field [8] and nonsymmetric gravitation theory (NGT) [9], predict deficits in the geodetic drift whose altitude signatures differ from PPN theories; and NGT predicts no change in frame-dragging.

This letter describes an orbiting gyroscope experiment which can measure the relativity drifts to an accuracy somewhere between 3 and 0.1 μas/yr and which can make a series of measurements at different altitudes. This corresponds to a measurement of γ to between 10^{-7} and 10^{-8}, and would be the most accurate determination of γ proposed to date. A value of γ less than one combined with the altitude dependence of Eq. (1) would indicate the existence of a massless scalar field, and a value of α_1 different from zero would imply a preferred reference frame. A non-PPN theory of gravity such as NGT or a light massive scalar field, if seen, could be uniquely identified. It would also be possible to measure the frame-dragging drift to about 10^{-5}. In this case, one could place an independent limit on the PPN α_1 of about 10^{-4} or concordantly verify a measurement of γ to 10^{-5}.

The experiment described here was developed at Stanford University between 1961 and 1972 and from 1989 until the present. It consists of a rapidly spinning drag-free satellite [10] at ambient temperatures chasing a pure unsupported gyroscope (USG) [10, 11]. The angular momentum vector of the gyro is read out by redundant two-axis autocollimators reflecting from 5-mm optical flats on the north and south poles of a spherical rotor, and the direction to the reference star is measured by two redundant two-axis telescopes. The difference of these two measurements is the angle between the gyro and the star. Since the satellite spins, all zero-point and satellite-fixed errors are at DC while the gyro-satellite and star-satellite angles are modulated at the satellite roll rate. Error sources such as unsymmetrical darkening or misalignment of the optics, electronic drifts, detector zero-point shifts, structural bending, temperature variations, etc. are only important if they have a component at the satellite spin rate. On the other hand, scale factor variations are not shifted away by the roll frequency but may be calibrated during the experiment from the aberration of starlight.

From 1961 to 1972 the performance of the drag-free satellite and the unsupported gyroscope were investigated, a three-axis drag-free satellite was flown [12], and a laboratory version of an autocollimator readout for the gyro was demonstrated [13]. This included the solution to the problems of fabricating the rotor and actively aligning the spin axis with the rotor maximum axis of inertia during initial spinup (active damping [14]). During the last four years the experiment has been refined to greatly improve the performance, extensive error calculations have been done, and a detailed design has been worked out [15].

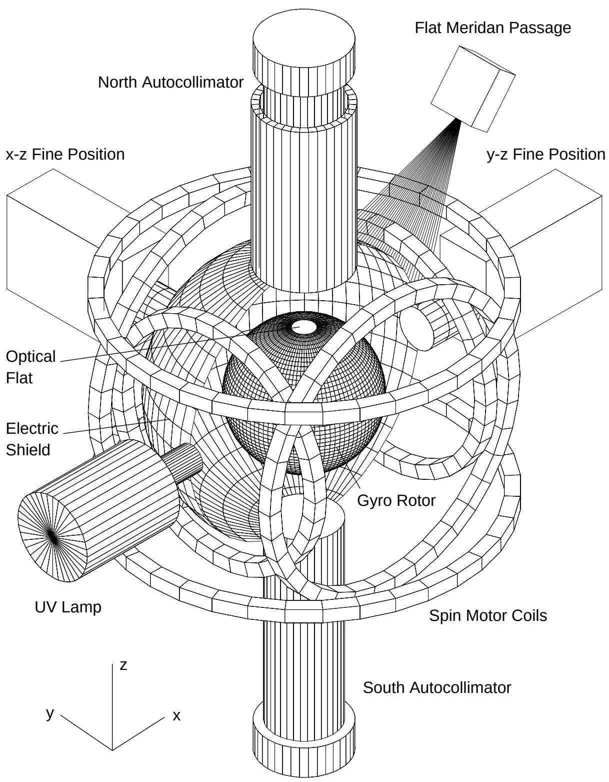

FIG 1. Gyro Rotor, Autocollimators, Spin-Motor Coils, Cavity Wall, Fine Position Sensors, UV Lamp, etc.

Figure 1 shows a schematic view of the 5-cm gyro rotor, housing, and instruments. The rotor is made of Silicon doped to 1,000 to 100,000 mhos/meter, and the three pairs of coils on orthogonal axes show the 16- to 160-kHz three-axis eddy-current induction motor which must perform three functions: spinup to about 900 Hz, active damping, and alignment to the star. The two-axis fine-position sensors are autocollimators with the exit beam focused to a point at the center of the rotor making them sensitive only to translation (transcollimators). The UV light source discharges the rotor. The hemisphere behind the rotor is a cut-away of the electric shield and vacuum can, and the array of lines in the far background is a fiber-optic bundle to detect the meridian passage of the flats for coarse active damping [14]. The electric shield is divided into three pairs of orthogonal electrodes for coarse position sensing and charge measurement. The gyro and instruments are enclosed in a multilayer Mumetal magnetic shield, and this assembly is enclosed together with the two telescopes in an insulating chamber. The satellite would carry sufficient nitrogen control gas for drag-free lifetimes of 10 to 25 years and sufficient monopropellant hydrazine for inclination changes of up to six degrees or equivalently for altitude changes up to 1500 km.

This configuration has a gyro drift error of about 0.05 μas/yr, gyro readout noise between 250 and 6 μas/Hz$^{1/2}$ depending on the autocollimator design, and a noise error with a 40-cm telescope of 300 to 5 μas/Hz$^{1/2}$ depending on the reference star [15, 16]. The low drift is achieved by: (1) Operating the gyro in the pure unsupported mode. All support related torques from atmospheric drag, solar radiation pressure, centrifugal force, zero-g offset, etc. are nonexistent; and the only electric fields in the cavity arise from rotor charging and surface-effect fields. (2) Using a wide gap of one cm so that surface-effect fields are small in comparison to those from rotor charging which can be controlled with the UV light. (3) Choosing the orbit to null the gravity-gradient drift at the end of each run. Acceptable drifts can be achieved with a rotor with $\Delta I / I = 10^{-4}$ greatly simplifying rotor fabrication. (4) Controlling the spin axis of the satellite to coincide with the spin axis of the gyro rather than with the star. This improves roll averaging by about 1000 since the two spin axes do not separate due to the annual aberration of starlight. Because of this, a vacuum of 10^{-8} to 10^{-9} torr at normal temperatures is sufficient to suppress drag-misalignment torques. Data from the September 1972 drag-free flight give experimental support for the 0.05-μas/yr drift error calculated for this system. The measured disturbing specific force was 5×10^{-12} g's compared to a design goal of 10^{-11} g's validating the methodology used to calculate the forces and moments on the rotor [10,12,15]. If this disturbance came only from surface forces, the equivalent gyro drift with a rotor sphericity of 10^{-6} and roll averaging of 10^{-7} would be about 5×10^{-7} μas/yr.

In addition to the inherently low noise equivalent angles of the autocollimator and the telescope, high readout accuracy is also achieved by using the aberration of starlight to cancel the accumulated angle between the star and the gyroscope. Cancellation inside of a band of ±0.2 or ±0.1 arcsec can be achieved once per orbit for a few days at the beginning and at the end of an experiment giving a total averaging time of several thousand seconds [15]. This reduces the dynamic range over which the telescope must work from ±50 arcsec and alleviates the scale-factor calibration problem by a factor of 250 to 500. Data would also be taken during the entire run, but the precision measurements would be made at the beginning and end of the experiment. After low noise instruments, measurement in null is the second major reason for high readout accuracy.

The most severe obstacle, however, to high accuracy is determining the proper motion of the reference star. There is some hope that VLBI measurements of radio stars can achieve accuracies of about 3 μas/yr with accurate modeling and observations over 20 to 40 years [17], and the astrometric programs POINTS and OSI may be able to measure the proper motions of selected stars to 0.4 μas/yr over the ten-year life of an astrometric flight if it takes place at the same time as the gyro experiment [18, 19]. It should be clear, however, that this will be difficult.

Fortunately there is a method of doing the experiment which does not depend on a prior measurement of the proper motion of the reference star. Since the eddy-current spinup motor can be reused it is possible to spin down the gyro, change the orbit, respin the gyro, and repeat the experiment. Furthermore because of their small size and light weight, it is possible to orbit two satellites on one launcher. This opens the possibility of making a series of simultaneous one-year measurements with two satellites at separate altitudes or, even better, of using nodal regression to place the satellites in counter-rotating orbits and then making simultaneous measurements of the geodetic drift using the same reference star. In either configuration, the two measurements would be subtracted eliminating the declination error in the star's position; and the one-year measurements would be repeated at different altitudes. It is this technique which opens the possibility of measuring the geodetic drift to an accuracy of 0.1 μas/yr with 10^4 seconds of averaging and 5- to 10-μas/Hz$^{1/2}$ instruments. For frame-dragging, this approach can also be applied to the proper motion in right ascension by using two satellites, one in a polar and one in an equatorial orbit; but unlike the geodetic measurement, it would require separate launches.

With cancellation, the aberration of starlight becomes the precision angle standard of the experiment. The main error comes from the satellite orbital velocity and is 2×10^{-5} m/sec giving an aberration error of 0.014 μas. This is the only error in a two-gyro experiment since the errors from the earth's motion are canceled out. In a single-gyro experiment, the largest error does not come from the earth's orbit but from galactic rotation and is 0.8 μas [15].

Thermal errors are controlled by rapidly spinning the satellite and placing the gyro, all optical instruments, and the analog electronics inside of an insulated chamber. Errors below 0.1 μas require that temperature gradient variations at roll rate not exceed 10^{-6} K/meter. With satellite spin rates of 0.1 to 1 Hz, one or two centimeters of insulation is sufficient to attenuate temperature waves through the walls by a factor of 10^{-12} or more. Besides the walls, temperature variations and electrical/mechanical roll coupling can enter through a number of paths. These error sources have been investigated [15], and the analysis shows that they can be controlled to 0.1 μas or less. The use of low temperatures [20] does not ameliorate this problem since a SQUID readout has a temperature sensitivity of about 10 arcsec/K compared to 0.1 arcsec/K/meter for well designed optics, and in any case the temperature sensitive analog electronics and gyro suspension circuits must be located outside of the dewar. The absolute temperature requirements are not severe, ±1 to ±10 K, because the satellite maximum-inertia axis is aligned with the autocollimator zero points by an automatic mass trim system [21], and the telescope zero points are servoed to ±0.1 arcsec. Finally, from the above paragraphs it can now be seen why it is possible to improve on GP-B by 10^2 to 10^3 once the constraint that it be done at low temperatures is removed.

If it is desired to cross-check the results by having multiple gyros in one satellite, the additional gyros must be electrically supported (ESGs). Extra gyros are undesirable because of the added complexity and size and because the ESGs place roll-rate and mass-distribution restrictions on the satellite degrading the performance. There are four better ways to cross-check the gyro drift: (1) Do a series of one-year experiments at different altitudes and determine the errors from the scatter in the data. (2) Do a cross-comparison between two satellites. (3) In an orbit which measures the geodetic and frame-dragging drifts separately, use the gyro axis orthogonal to the relativity drift to do a check which is 20 to 50 times more accurate than with ESGs. (4) Design the gyro so well that a cross-check is not critical. The USG drift is as much as 3000 times less than that of an ESG, and there are only about 23 drift sources for a USG compared to about 70 for an ESG so that it is much easier to be sure that the list is exhaustive.

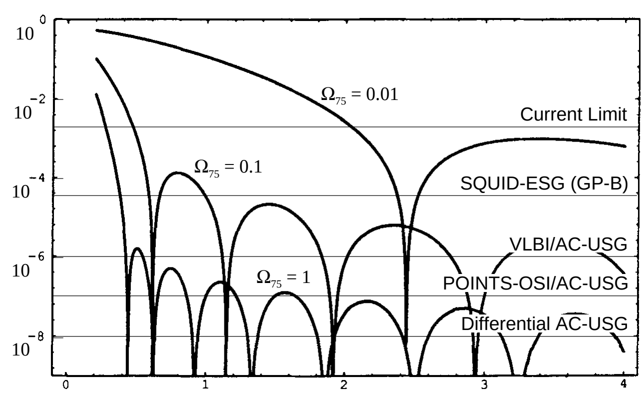

FIG 2. Damour-Nordtvedt prediction of $(1-\gamma)/(1-\gamma)_R$ versus κ, the curvature at a minimum of the potential defined by the coupling coefficient. The horizontal lines are the limits placed on $1-\gamma$ by past and future experiments drawn for the case $(1-\gamma)_R = 1$ (adapted from Ref. [7]).

A comparison of experiments is shown in Fig. 2 which is adapted from [7]. It shows $(1-\gamma)/(1-\gamma)_R$ or equivalently ω_{BDR}/ω_{BD} where R is the value at the end of the radiation era. The abscissa is the curvature of a minimum of the potential defined by the scalar-tensor coupling coefficient (see Ref. [6, 7]). $(1-\gamma)_R$ is expected to be of order unity although it could be as small as 0.1 or less. The curves are drawn for three values of the average density of the universe assuming $H_0 = 75$ km/sec/Mpc. The horizontal lines have been added to show the limits which have been placed or will be placed on $1-\gamma$ by various experiments. They are drawn for $(1-\gamma)_R = 1$; and if $(1-\gamma)_R$ were 0.1, they would have to be shifted up by one order of magnitude. The line labeled 'Current Limit' is the present limit in $1-\gamma$ given by radar time delay and VLBI deflection experiments [5], and 'SQUID-ESG (GP-B)' is the limit which would be placed by the currently planned Gravity Probe B experiment [20]. 'AC-USG' stands for Autocollimator-Unsupported Gyroscope and is the experiment described in this letter. 'VLBI/AC-USG' means that a radio star's proper motion has been previously determined to 3 μas/yr, and 'POINTS-OSI/AC-USG' gives the limit if either of these programs flies and determines proper motion to 0.4 μas/yr. 'Differential AC-USG' represents the accuracy that could be achieved using the technique of simultaneous measurements from two experiments in counter-rotating orbits.

The importance of high accuracy is clearly shown in Fig. 2. For example for $\Omega_{75} = 1$, 10^{-7} is needed to have a good chance of seeing a finite value for $1-\gamma$. If on exiting from the radiation era $(1-\gamma)_R$ were very much less than one, the required accuracy would become 10^{-8} or better. But even if $1-\gamma$ were large, high accuracy would still be important for a concordant frame-dragging measurement.

A fuller treatment of the scientific significance of a high accuracy version of the gyro experiment is given in [22] which includes a discussion of a recent paper by Damour and Polyakov [23]. This paper applies the theory of [6, 7] to the massless string dilaton. Ref. [22] shows that with just the right parameters, the string mass scale might be measured within the context of this theory.

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