Managing Spherical Proof Masses in Drag-Free Satellites with Application to the LISA Experiment

Benjamin Lange
1922 Page St.
San Francisco, CA 94117-1804
Tel. and Fax +1-415-221-6600, Ext. 310
email: blange@virtualpbx.com

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Abstract

The very low specific-force noise specification of the LISA mission requires a drag-free satellite for its realization. Not only is the lowest specific-force noise most likely to be achieved by a completely free-floating proof mass with a wide gap and no applied forces or torques, but the performance of such a system can be assured by prior drag-free satellite experience so that a technology demonstration mission for the drag-free specification would not be needed. Better performance can be achieved if it is possible to present a defined reflecting surface to the laser beam. This paper discusses the techniques of having a defined surface and other issues involved with spherical proof masses.

Introduction

LISA is a deep-space mission designed to detect gravity waves by measuring the change in distance between three sun-orbiting spacecraft in a equilateral triangle 5 million kilometers on each side forming a laser interferometer. In order to prevent the distance changes between the satellites from being excessively disturbed by local forces acting on the them, it is necessary that the three LISA satellites be drag-free [1, 2]. One role of the satellites is to shield the proof masses to the maximum extent possible from all disturbances. When this is done, the proof masses can be used as the mirrors in the three-arm interferometer and define the spatial geometry of the configuration.

There are two possible shapes for the mirrors, flat and spherical. A flat reflecting surface would be in the form of a cube reflecting a parallel beam, and a spherical reflecting surface would be a sphere reflecting a converging beam focused to the center of the sphere.

A cube suffers from the problems that torques must be applied to keep one of the cube's surfaces perpendicular to the beam and there is no known way to guarantee that the torques do not apply specific forces which exceed the allowed specification for the proof masses. In addition the most practical way to implement a cube is to use a single-axis drag-free satellite; and this means that in addition to the control torques, control forces must be applied along the two axes perpendicular to the beam worsening the disturbing force problem. Also for LISA, two cubes require that the satellite be designed with two Zero Self-Gravity-Gravity points.

In the case of a spherical proof mass, the attitude control problem can be solved in one of two ways, neither of which requires that forces or torques be applied to the proof mass during the experiment runs, i.e. after the initial spin-up phase. The Gravity-Probe B program has shown that spheres can be polished to an accuracy of about 0.8 microinch, i.e. about 20×10^{-9} meters. Since the LISA spacecraft will be separated by 5×10^{9} meters, a typical strain from a gravity wave of 10^{-21} to 10^{-23} [3] will produce a path variation of about 10^{-12} to 10^{-14} meters. Surface errors of a free-floating sphere would cause distance variations 20,000 to 2×10^{6} times this great, but if the sphere were rotating and polhoding about an arbitrary axis with a spin period shorter that 10 seconds, these disturbances would be out of the planned sensitivity band of LISA of 10^{-4} to 0.1 Hz.

An alternate approach would be to accurately calibrate the surface and to have it rotate stably about its maximum axis of inertia with the spin axis parallel to the yearly axis of rotation of the LISA plane. This would present a defined surface to the laser beams which could provide a calibration signal for the errors of the spherical surface. Notice that if the spin period were shorter than 10 seconds, this second approach also removes the disturbances from the sensitive band as well as providing the calibrated surface.

In both of the above cases the spin rate of the sphere would be between one and 10 Hz, but it might turn out that a dynamic range of 20,000 to 2×10^{6} would be difficult to achieve.

One way of reducing the dynamic range requirement would be to polish a better sphere. Almost all spheres which have been made up to now have been measured with Talyronds. A measuring system using an electric bearing and a transcollimator (discussed below) could potentially measure the quality of a sphere to 10^{-10} to 10^{-12} meters. A general rule of thumb is that if you set a specification, it can usually be met if there is a way to measure it. If this were to hold true for the selective polishing of the spheres, it might be possible to make a sphere accurate to 10^{-10} meters which would reduce the dynamic range requirements to between 100 and 10^4.

Given these considerations, the question then arises as to how accurately the sphere can be set spinning about its maximum axis and how accurately the surface can be calibrated.

Measuring the Principal Axes of Inertia of an Accurately Polished Sphere

At first glance it might be thought that a high quality solid sphere has very ill defined principal axes of inertia since the three principal moments of inertia are almost identical. It turns out, however, that the principal axes are very well defined, can probably be measured to a fraction of an arcsecond, and can provide the stable reference needed to give a well defined surface after calibration. The reason for this is that although body-fixed external torques can easily displace the measured principal axes, the very round surface of the sphere makes it difficult to apply disturbing torques to the body. In a spinning spherical rotor in a laboratory test setup consisting of an air or electric bearing, the body-fixed angular velocity vector polhodes in a ellipse when it is close to either a maximum or a minimum principal axis of inertia. The elliptical path surrounds the principal axis which is located at the center of the ellipse. If there are body-fixed disturbing torques present, the ellipse is displaced, and what is measured is an apparent principal axis. Spin-up torques can be applied to the rotor either in the laboratory or in space with a three-axis eddy-current induction motor. This requires that the rotor be at least slightly conducting (about 10^5 mhos/meter, 10^{-3} ohm-cm or better).

Let the body-fixed *x*-axis be parallel to the minimum axis of inertia and the *z*-axis be parallel to the maximum axis. The principal-axis measurement error can be calculated by solving Euler's equations with constant disturbing torques for a rigid body with principal moments of inertia, $I_x < I_y < I_z$. Define $\varepsilon_x \equiv (I_z - I_y)/I_x$ and $\varepsilon_y \equiv (I_z - I_x)/I_y$.

In a good sphere the $\varepsilon_{x,y}$ are about 10^{-5} to 10^{-7} depending on the material homogeneity and the degree of selective polishing. Under the assumption that the z-angular velocity, ω_z, is constant, the angle errors, ϕ_x and ϕ_y, caused by body-fixed disturbing torques, M_x and M_y, are given by

$$\phi_{x,y} = -\frac{M_{x,y}}{\varepsilon_{x,y} I_z \omega_z^2} \tag{1}$$

The disturbing moment can be estimated from the weight of the rotor and the distance that its center of mass is offset from its center of support. If the average radius of the sphere is a, let this offset be $a\varepsilon_{surf} \approx$ maximum radius minus the minimum radius. With this definition of ε_{surf}, $\phi_{x,y}$ is roughly given by

$$\phi_{x,y} \approx \frac{5\varepsilon_{surf} g}{2\varepsilon_{x,y} a \omega_z^2}. \tag{2}$$

In the case of a well polished sphere $\varepsilon_{surf} \approx \varepsilon_{x,y} / 2$, and the two ε's cancel in the numerator and denominator so that the accuracy of a principal-axis measurement is roughly independent of the quality of the sphere. In an air bearing where the proof mass can be rotated at about 3000 RPM, the Equation 2 gives about 0.5 degree for the principal-axis measurement error; and in an electric bearing where 40,000 RPM is possible, the error is about 8 arcseconds.

For several reasons it is desirable to polish small optical flats at the north and south poles of the rotor, i.e. at each end of the z-axis. In this case the rotor is essentially symmetrical, and $\varepsilon_{x,y} \approx \varepsilon \approx 15(d_{flat} / d_{rotor})^4 / 16 \approx 10^{-4}$ for a flat with a diameter of ten per cent of the rotor diameter. In this case the increased moment-of-inertia difference ratio improves the principal-axis measurement accuracy by about a factor of 100, i.e. about 20 arcseconds in an air bearing and about 0.1 arcseconds in an electric bearing.

Thus it can be seen that it is possible to use the principal axes to fix a coordinate system in a high quality sphere with great accuracy. Two practical considerations not discussed here are thermal expansion and elasticity, but these do not change the essential results. In space thermal expansion is not a problem since thermal expansion cannot rotate the principal axes of a body with a homogeneous coefficient of thermal expansion. On the ground in a laboratory, thermal expansion can move the center of mass relative to the center of support in a bearing; so that the temperature of the rotor must be measured and taken into account. Elasticity is also not a problem in a body with homogeneous elastic properties. [4] and [5] report on air-bearing experiments for measuring the principal axes and give more details on thermal expansion and elasticity.

Polishing Optical Flats on a Spherical Proof Mass

It is useful to polish optical flats on each end of the maximum axis of inertia for three reasons:

- as seen above it improves the accuracy with which the principal axes can be defined,
- with the caveat that the rapid rotation of the LISA plane with respect to the normal to the ecliptic might create dynamic range problems, the proof mass can be used as a gyro to filter two of the three axes of the attitude reference signal from the laser beams and thus reduce the measurement noise in the attitude control system, and
- the flats can provide a reference signal during initial spinup to actively damp the rotor angular velocity to the maximum axis of inertia or to command a large polhode motion for surface calibration.

The basic technique for polishing the flats on the sphere is to measure the location of the maximum axis to better than one degree, polish an initial pair of flats, remeasure the maximum axis of inertia, and iteratively polish more accurate flats until the normal to the flat is parallel to the maximum axis to better than one arcsecond (i.e. within the linear range of an autocollimator).

In a perfect sphere, the initial optical flats could be arbitrarily placed and would then define the principal axes; but in a real sphere the natural $\varepsilon_{x,y}$ is large enough to cause an unacceptable error if the flat normal and the initial maximum axis are not aligned. Thus the natural maximum axis of inertia must be located before the initial flats are placed on the sphere. The maximum axis can be located to an accuracy of one or two degrees by placing the rotor in an air bearing with an eddy-current or air-jet spin motor. The spin axis is fixed in the laboratory but moves in the rotor (polhodes). The polhodes can be crudely traced by using a felt pen to put an ink dot on the spin axis of the rotor every few seconds. At the end of this procedure the polhodes will be traced on the sphere by the dots. (See the frontispiece in [6].) Next a grid can be scratched on the surface at the maximum axis and the rotor placed in a air bearing under a microscope with double illumination. One light, an ordinary white incandescent lamp, generates white circles from surface irregularities. The centers of the white circles show the spin axis. The second light, a strobe flashing at the spin frequency, shows the terrain of the surface where the spin axis is located. The spin axis will appear to move on the surface as it polhodes, and the maximum axis is to the right of the motion of the surface (i.e. to the left of the motion of the spin axis which circles the maximum axis in a counter-clockwise direction). The rotor may be damped by a manual active damper or by a soft brush until the polhode is small enough to locate the maximum axis to about one arcminute. For the details of this procedure see [7], and for the active damper see [8].

Once the natural maximum axis has been located to about one arcminute, initial flats can be polished on the rotor; and the rotor placed in an air or an electric bearing with autocollimator readout of the flat error [4]. The error can be measured, and the flats iteratively polished until the desired accuracy is achieved [4, 5]. A more detailed summary of this technique is given in [9].

Calibrating the Surface of the Sphere

The position of the surface of a sphere can either be read out by a capacitive pickoff, a fiber-optic laser interferometer, or by a transcollimator. This analysis will be based on a transcollimator readout. A transcollimator is a two-axis position sensor for a sphere. It is identical to an autocollimator except the output beam is focused at the center of the sphere. When this is done, a translation of the sphere, x, perpendicular to the beam is equivalent to a rotation, θ, of a mirror below an autocollimator given by $\theta = x / a$. Based on prior autocollimator theory and experiment, it is believed that a two-axis transcollimator could have a noise equivalent translation as low as 10^{-12} meters/Hz$^{1/2}$ $\times$ $(1\ \mu\text{watt} / W)^{1/2}$ where W is the power in the photon beam.

If a large polhode is commanded in a spherical proof mass spinning either in a drag-free satellite or in a laboratory electric bearing so that the transcollimator scans the entire surface, the surface errors of the sphere can be calibrated. The accuracy of this calibration might be as good as 10^{-12} meters although a more conservative estimate would be 10^{-10} meters, i.e. about one atomic distance. Calibration in space would have no bearing errors, and there now exists well tested theory for the torques acting on a sphere in an electric bearing making laboratory calibration also possible.

In a LISA satellite after calibration was complete, the active damper could drive the spin axis to the maximum axis of inertia; and the sphere would present a known surface to reflecting beams striking its equator to an accuracy of about 10^{-10} meters and perhaps as much as 10^{-12} meters.

Assuring the LISA Specific Force Requirement in the Sensitive Band

Prior drag-free satellite experience with the DISCOS satellite which was flown in 1972 has demonstrated a technique for assuring the LISA in-band specific-force requirement of 3×10^{-15} meters/sec^2/Hz$^{1/2}$ in the band of 10^{-4} to 0.1 Hz. DISCOS demonstrated a DC disturbing specific force of less than 5×10^{-11} meters/sec^2 and a gravity gradient of less that 10^{-7} /sec^2 [11]. The largest disturbances were calculated to come from the satellite self gravity, and the above performance was accomplished by calculating the gravity and gravity gradient of every mass in the satellite before the flight to make sure the self gravity and its gradient were within the specification of 10^{-10} meters/sec^2 [12]. In addition the satellite was designed to limit other sources of disturbing specific force to be below this level. The performance was verified by tracking the orbit.

Given that no field gradient at the DISCOS proof mass exceeded 10^{-7} / sec^2 and assuming that the LISA drag-free specification of 10^{-9} meters/Hz$^{1/2}$ can be achieved, the LISA proof-mass disturbances due to satellite motion would be less than 10^{-16} meters/sec^2/Hz$^{1/2}$. The other two categories of disturbance, external effects such as the interplanetary magnetic field or radiation and internal disturbances which do not depend on satellite motion such as collisions from the residual vacuum, are well in hand and should present no major performance problems. Complete calculations of the LISA disturbances for a spherical three-axis free-floating proof mass were presented at the Amaldi conference in Australia and will be the subject of a future paper. (See also www.dragfreesatellite.com, Recent Papers.)

The important point is that the DISCOS experiment has measured the most important disturbance source and shown that it can be limited below the LISA specification.

Furthermore the DISCOS result shows that the technique of determining the DC gravity and gravity gradient at the proof mass by prior calculation works.

Summary

A spinning calibrated spherical proof mass reflecting laser beams with the proof-mass spin axis parallel to the yearly axis of rotation of the LISA plane can be realized with sufficient accuracy to meet the LISA requirements. Spherical proof masses have several advantages over cubes:

1) No forces or torques need be applied to the proof mass and a wide gap (1 cm) can be used..
2) The LISA specific-force specification can be assured.
3) A technology-demonstration flight for the drag-free performance would not be needed.
4) The proof mass surface errors can be calibrated.
5) Using multiple transcollimators, a spherical system can potentially be more reliable.
6) The attitude control of the satellite can possibly be improved.
7) The satellite need be designed with only one Zero Self-Gravity-Gradient point.

Based on these considerations, LISA design studies should begin with spherical proof masses and only consider cubes if practical problems force it, not the reverse. In any event both spheres and cubes should be included in the initial studies.

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