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The Scientific Significance of a Long-Lifetime High-Accuracy Version of the Relativity-Gyro Experiment With Altitude-Change Capability

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Abstract: Recently three papers have calculated the expected deviation of the Eddington or PPN parameter, γ, from unity caused by the existence of a massless Jordan-Brans-Dicke field or a massless string-dilaton field. An orbiting relativity-gyroscope experiment which can measure γ to an accuracy of 10^{-7} or 10^{-8} has a chance of detecting one of these fields if they exist. A series of one-year experiments at various altitudes increases confidence in the results, measures the experiment errors, and provides a unique interpretation of the data.

In a separate paper [1], an ambient temperature orbiting gyroscope experiment is described which can measure the relativistic gyro drift with respect to the distant stars to an accuracy between 3 and 0.1 μas/yr depending on a number of design factors. This represents an improvement of 10^2 to 10^3 over the currently planned GP-B experiment and is accomplished primarily by using an autocollimator to read out the gyro spin direction. In addition to high accuracy, operation at room temperature allows the experiment to be designed to last between 10 and 25 years; and a series of one-year experiments can be repeated in altitude increments of 100 km. This paper will address the question of what can be learned about the theory of gravity from such an experiment.

Classical general relativity (GR) predicts space-time singularities and, when quantized, does not lead to a renormalizable perturbation theory. Thus it is not generally believed to be the exact theory of gravity. Many of the possible alternatives are encompassed in the *parameterized post-Newtonian* (PPN) frame work [2]. This formulation includes the Jordan-Brans-Dicke (JBD) or scalar-tensor (ST) theories [3]. As first pointed out by Pascual Jordan in a 1944 paper in the *Physikalische Zeitschrift* which was destroyed by the war [4], Kaluza-Klein theories, when taken in their most natural form, predict the existence of an additional scalar source of the gravitation field which is conventionally interpreted as being G or $1/G$. More recently, it has been known for about two decades that super-string theory predicts a scalar partner of the graviton known as the dilaton [5]. In addition, scalar-tensor inflation terminates more naturally than inflation based on general relativity [6], and scalar fields are ubiquitous in extensions of the standard model. Thus, scalar-tensor theories are by far the most serious candidates for an alternative to general relativity; and this paper will concentrate almost exclusively on the relevance of the gyro experiment to them.

Scalar-tensor theories in the Jordan-Fierz (physical) metric are characterized by a coupling parameter, ω. When ω equals infinity, they go over into general relativity. ω is connected to the Eddington or PPN parameter, γ, by $\gamma = (\omega+1)/(\omega+2)$ or, when ω is large, by $1/\omega \approx 1-\gamma$. The average drift of a perfect gyro in orbit around a spherical earth with no other astronomical bodies present is predicted by all PPN theories to be [2]

$$\langle\dot{\phi}\rangle = \left[\gamma + \frac{1}{2}\right]\frac{(M_\oplus G)^{3/2}\hat{\mathbf{h}}}{c^2 r_S^{5/2}} + \left[\frac{\gamma+1}{4} + \frac{\alpha_1}{16}\right]\frac{(\mathbf{J}_\oplus - 3\hat{\mathbf{h}}(\hat{\mathbf{h}}\cdot\mathbf{J}_\oplus))G}{c^2 r_S^3}.$$

$\hat{\mathbf{h}}$ is the unit orbital angular momentum vector, $\mathbf{J}_\oplus$ is the earth's angular momentum vector, r_S is the distance from the center of the earth to the satellite, and α_1 is a PPN parameter which allows for the existence of a preferred reference frame. In general relativity $\gamma = 1$ and $\alpha_1 = 0$. Measurements in the solar system and in binary pulsar systems have shown

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that $|1-\gamma|$ is less than 2×10^{-3} (i.e. $|\omega|$ is greater than 500) and $|\alpha_1|$ is less than 10^{-4} [2]. The gyro experiment described in [1] can measure γ to between 10^{-7} and 10^{-8}.

PPN theories are not the only possibilities for a more exact theory. Two examples are nonsymmetric gravitation theory (NGT) [7] and the theory of a light massive scalar field [8]. They predict a geodetic precession slightly less than the PPN theories, and this geodetic deficit also has a different altitude signature than the PPN prediction, $r_S^{-11/2}$ for NGT and $(1 + r_S/\lambda)\, e^{-r_S/\lambda} / r_S^{5/2}$ for a light massive scalar field with range, λ. These two examples are included merely to show non-PPN theories from the recent literature with a different altitude dependence for the gyro drift than the PPN theories. They both have difficulties which make it somewhat unlikely that they would turn out to be true. NGT has recently been severely criticized [9], and solar system measurements have almost excluded it [10]. The possibility of a light massive scalar field has been strongly restricted by the fifth-force experiments [8], and any field mass greater than about one eV would be indistinguishable from general relativity. The range of interest, 2,000 to 200,000 km, corresponds to a mass between 10^{-13} and 10^{-15} eV; and it is not clear why a finite mass would be so small. Furthermore, orbit tracking experiments using a drag-free satellite would be a more sensitive test of the existence of a fifth force in the above range than a gyro experiment. Nevertheless, the gyro must be able to detect non-PPN theories if they exist.

For this reason a uniquely interpretable scientific result requires that the experiment be performed versus altitude. A series of measurements at different altitudes repeats the experiment giving high confidence in the results and determines the experiment errors from the scatter in the data . Since all PPN theories have the same altitude signature, an incorrect altitude dependence would indicate a non-PPN theory of gravity. The converse is not true as shown by the massless dilaton theory which will be discussed shortly.

When solar system experiments showed $\omega \geq 500$, the JBD theory was no longer considered to be viable. Two papers [11, 12] have now been published which show this to have been premature. They demonstrate that the scalar-tensor theories contain a mechanism which causes them to tend to general relativity as the universe ages. $(1-\gamma)/(1-\gamma)_R$ where R indicates the end of the radiation era has been calculated and is shown in Fig. 1a which has been taken from [1] and [12] . κ is the curvature parameter of a parabolic fit to the minimum of the potential function which determines the coupling parameter [11, 12]. The curves are shown for three different values of the density parameter Ω_{75} with $H_0 = 75$ km/sec/Mpc. The horizontal lines indicate the present measured error of $1-\gamma$ and the accuracy with which various proposed gyro experiments will improve it. They have been plotted under the assumption that $(1-\gamma)_R = 1$. If $(1-\gamma)_R$ were less than one, the horizontal lines would have to be shifted upwards. 'AC-USG' stands for Autocollimator-Unsupported Gyroscope and is the technology described in [1]. 'VLBI' assumes that the proper motion of the reference star is measured to 3 μas/yr using the VLBI network, and 'POINTS-OSI' indicates a proper-motion accuracy of approximately 0.4 μas/yr from a future satellite astrometric program. 'Differential AC-USG' refers to a special technique described in [1] which can determine the proper motion to 0.1 μas/yr without prior astrometric measurements. The main point of Fig. 1a is that high accuracy is essential to having a chance of detecting a massless scalar field.

The value of $(1-\gamma)_R$ is constrained by nucleosynthesis, but a companion paper in this volume [13] shows that $(1-\gamma)_R \approx 1$ is still allowed. This may also be seen as follows: The main constraint comes from He^4 synthesis where with a changed Hubble constant, H_{NS}^{ST}, the fraction of He^4 is increased by $0.142 \ln x$ for $x \equiv H_{NS}^{ST}/H_{NS}^{GR}$ [14]. Experimental uncertainties limit this change to about 0.01, and the sensitivity to x implies that $x \leq e^{0.07}$. Using the notation and results of [12], $x = A_R / A_0\, (1 + \alpha_0^2)^{1/2} \approx e^{a_R - a_0} = e^{(\alpha_R^2 - \alpha_0^2)/2\kappa}$. This in turn implies that $(1-\gamma)_R \approx 2\alpha_R^2 < 0.07\times 4\kappa = \kappa / 3.6$. $(1-\gamma)_R$ could, however, be as small as 0.1 or 0.01 further emphasizing the need for high accuracy as all the horizontal lines in Fig. 1a would then be shifted up by 10 or 100.

The results presented in [11,12] and Fig. 1a assume that the equivalence principle is satisfied exactly and that the scalar-tensor theories are correct. A more fundamental approach is to look at the dilaton without any assumptions about the equivalence principle. The problem has been that no mechanism has been found to give the dilaton the mass deemed necessary to avoid a number of effects which are not seen experimentally. Recently, this has been viewed from another direction [5]. It is assumed that the dilaton remains massless but is universally coupled to matter and radiation so that the attractor mechanism of scalar-tensor theories toward GR causes the as-yet-unseen effects to be very small.

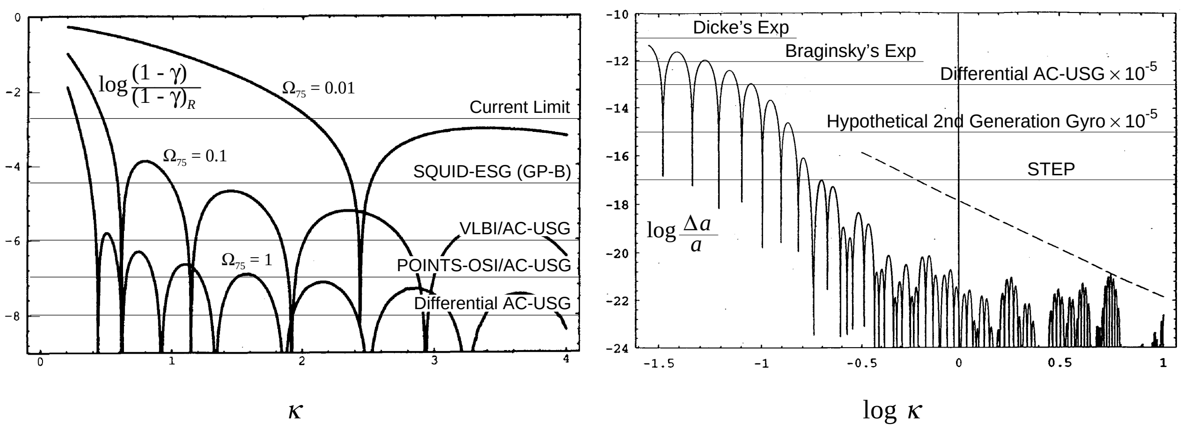

FIG 1a. Damour-Nordtvedt prediction of $(1-\gamma)/(1-\gamma)_R$ versus the curvature, κ. The horizontal lines are the limits placed on $1-\gamma$ by past and future experiments for the case $(1-\gamma)_R = 1$ (adapted from Ref. [12, 1]). FIG 1b. Damour-Polyakov prediction of the violation of the equivalence principle by a massless dilaton as a function of the curvature parameter, κ. Adapted from [5].

Fig. 1b shows the violation of the equivalence principle, $\Delta a/a$, caused by a massless dilaton field as a function of the same curvature parameter as in [11, 12] and is adapted from Fig. 3 in [5]. The question may be asked what is the relevance of Fig. 1b to a gyro experiment? The answer is that in massless-dilaton theory, the violation of the equivalence principle and $1-\gamma$ are connected. Using the notation and results of [5]

$$\frac{\Delta a/a}{1-\gamma} = \left[\frac{\partial\hat{\sigma}}{\partial \ln B^{-1}}\Delta\left(\frac{N+Z}{M}\right) + \frac{\partial\hat{\delta}}{\partial \ln B^{-1}}\Delta\left(\frac{N-Z}{M}\right) + 8\times 10^{-4}\Delta\left(\frac{E}{M}\right)\right] \Bigg/ \left[1 + 2\ln\left(\frac{\Lambda_S \mu_A}{u_3}\right)\right]$$

where in an atom of the test mass, N is the number of neutrons, Z the number of protons, M the atomic mass, and E the coulomb binding energy. $B(\phi)$ is the universal coupling parameter of the dilaton ϕ-field, Λ_S is the string mass scale assumed to be 3×10^{17} GeV, u_3 is the QCD mass scale taken as 0.931 GeV, and μ_A is a dimensionless parameter of order unity so that the ln term is equal to 40.3. The first two terms in the numerator are assumed to be negligible, and the Δ functions indicate taking the difference for the atoms of the two proof masses in an equivalence-principle experiment.

As an example, the ratio E/M for Be_9^4 is 0.9, Si_{28}^{14} 2.1, Cu_{63}^{29} 3.2, and U_{238}^{92} 5.7; so that an experiment comparing copper and silicon gives $\Delta a/a \approx 10^{-5}(1-\gamma)$. This means that if the equivalence principle has been tested to 10^{-12}, then γ has been measured to 10^{-7}. If then γ has already been tested to one part in 10^7, why fly a gyro experiment?

Besides the obvious answer that [5] is a theory, there are four further reasons to do both a high-accuracy gyro experiment and a high-accuracy equivalence-principle experiment: 1) The two experiments are independent and measure different aspects of gravitational theory, 2) Measuring both $\Delta a / a$ and $1 - \gamma$ measures the string mass scale within the context of the Damour-Polyakov theory, 3) $\Delta a / a$ may not be as small as Fig. 1b would indicate due to effects such as the possibility, for example, that $B(\phi)$ may be slightly different for each dilaton coupling, and 4) A second generation gyro experiment may be able to improve the accuracy as much as two orders of magnitude to measure γ to 10^{-10}. From this point of view, an additional reason for flying the gyro experiment, would be to gain the necessary knowledge to fly a second higher accuracy one.

Should $1 - \gamma$ turn out to be greater than about 10^{-6}, there would be an additional advantage of a high-accuracy gyro experiment, a corresponding reduction in the frame-dragging drift could be seen; and this would give a concordant verification that the value of γ as measured by the geodetic drift was real. In general, a direct measurement of frame-dragging is less valuable within the PPN framework than a geodetic one since it must achieve an accuracy of at least 10^{-5}, the current limit on $\alpha_1 / 4$, to provide new information.

In conclusion a high accuracy gyro experiment has a chance of detecting a massless scalar field, and the high accuracy combined with measurement versus altitude would improve the believability and uniqueness of the results. Conversely, a single measurement of γ which did not determine the variation with altitude and whose accuracy was too low to show a concordant reduction in frame-dragging would be unable to distinguish between a massless scalar field, a long range scalar field, a non-PPN theory of gravity, some other reason for γ being less than one, a common-mode drift, or a common-mode readout error.

The effect of a scalar field in the present era is minimal, but it has consequences for early-universe astrophysics since ST attractor theories imply that it was once much larger. Extended inflation [6] provides an example of the effect of even a small deviation from GR illustrating the importance of experiments to determine a more correct theory of gravity.

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