

Expected Electric Charging Rates and Disturbances In a Two-Sphere Free-Fall Equivalence-Principle Experiment in a Drag-Free Satellite

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One design for a high precision Equivalence-Principle (EP) experiment would be a drag-free satellite [1] with two free-floating concentric spherical proof masses, a solid sphere inside of a spherical shell. The question has been raised whether or not it is possible to measure the electric charge on a conducting sphere inside of a closed conducting spherical shell. This paper presents a solution to this problem and, in addition, discusses the expected charging rate of the proof masses. The question of proof-mass charging in orbit, although the subject of several theoretical papers and two space experiments, has by no means been settled.

Apart from disturbances such as electric charge, there is a fundamental problem with any Equivalence-Principle experiment in space: an EP violation cannot be distinguished from a semimajor-axis error between the proof masses. Furthermore for a free-fall experiment there is a further problem, the deviation due to an EP violation only grows as t not t^2 . These two problems can both be solved by a technique known as DC Cancellation [2]. This consists of placing two spherical gravity-gradient cancellation masses on each side of the proof masses in a satellite spinning perpendicular to the orbit plane with the line connecting the cancellation-mass centers also perpendicular to the orbit plane. Define x to be in the radial direction, y to be parallel to the orbit velocity, and z to be perpendicular to the orbit plane. Using this definition, the cancellation masses can be chosen so that their gravity-gradient tensor at the proof masses takes the form $\text{diag}[-3n^2 \quad -3n^2 \quad 6n^2]$ where n is the mean orbit rate. With this additional gravity-gradient, the classical Euler-Hill equations of relative orbital motion are transformed by the cancellation masses as

$$\begin{array}{ccc} \ddot{x} - 3n^2x - 2n\dot{y} = f_x & \xrightarrow{\text{DC Cancellation}} & \ddot{x} - 2n\dot{y} = f_x \\ 2n\dot{x} + \ddot{y} = 0 & & 2n\dot{x} + \ddot{y} + 3n^2y = 0. \end{array}$$

The solutions with an initial x_0 and an EP violation, f_x , are then transformed as

$$\begin{array}{ccc} x = x_0 + (f_x + 3n^2x_0)(1 - cnt)/n^2 & \xrightarrow{\text{DC Cancellation}} & x = x_0 + \frac{3}{14}f_x t^2 + \frac{4f_x}{49n^2}(1 - c\sqrt{7}nt) \\ y = -2(f_x + 3n^2x_0)(nt - 5nt)/n^2 & & y = -\frac{2f_x}{7n}t + \frac{2f_x}{7\sqrt{7}n^2}S\sqrt{7}nt \end{array}.$$

The solution of the original Euler-Hill equations shows the two problems mentioned above. The term, $f_x + 3n^2x_0$, in the y -solution shows for example that an error in the x -direction of only 3×10^{-12} meters would mimic an EP violation, $\Delta a/a$, of $10^{-18} g$. The term, nt , shows that the free-fall response to an EP violation only grows as t , whereas the largest disturbance, brownian motion due to the imperfect vacuum, grows as $t^{3/2}$.

The solution to the transformed equations shows that in the ideal case, the $3n^2x_0$ term no longer appears with f_x ; and the $\frac{3}{14}f_x t^2$ term shows that the DC-canceled EP-violation response grows as t^2 .

In the practical case where there is a cancellation error, k^2 , the combination $f_x + k^2x_0$ replaces f_x in the transformed solutions; and the t^2 response lasts between 10^4 and 10^6 seconds depending on k^2 . The term $f_x + k^2x_0$ means that the non-observability problem is suppressed by the ratio, $k^2/3n^2$.

Once the radial-nonobservability and growth-only-as- t problems have been solved, one of the largest error sources is electric charge on the proof masses.

The electric force from charges, q_1 and q_2 , on the conducting sphere and shell can be calculated by differentiating the expression for the stored electrostatic energy with respect to the miscentering. By bringing a small test charge in from infinity, it can be shown that the elastances are given by $s_{11} = 1/c_{11} - 1/c_{23} + 1/C_3$, $s_{12} = s_{21} = s_{22} = -1/c_{23} + 1/C_3$ and $s_{13} = s_{23} = s_{31} = s_{32} = s_{33} = 1/C_3$ where $c_{11} = c_{11S}(a, b, x_{ce})$, $c_{23} = -c_{11S}(c, d, x_{ce})$, c_{11S} is the expression for c_{11} in [3], the arguments are the radii and the miscentering x_{ce} of the sphere-shell and shell-cavity surfaces, and C_3 is the total capacity of the satellite. It is possible to use the theory from a two-sphere system for a three-sphere nested system because of the shielding provided by the shell, and the relevant electrostatic energy is $W = \frac{1}{2}q_1^2/c_{11} - \frac{1}{2}(q_1 + q_2)^2/c_{23}$ so that $F_1 = \frac{1}{2}(q_1/c_{11})^2 dc_{11}/dx_{ce}$ etc.

The charge on the outer shell can be measured by the method used with GP-B [4], i.e. by applying a DC electric field and measuring the electric suspension force necessary to compensate it. The charge on the inner sphere can be measured by forcing an oscillatory motion of the shell with the cavity electrodes and observing the resulting motion of the sphere. Calculations using the above expression for the electric forces show that the motion of the sphere can be detected with sufficient accuracy to measure the electric charge and suppress it so that an EP experiment of 10^{-20} g or better is possible.

The only known method of proof mass charging by the space environment is from 100 to 200 MeV protons either from solar flares or from trapped protons principally in the South Atlantic Anomaly (SAA). This result is supported by two theoretical papers [4, 5] and by the measurements of the Cactus experiment flown by ONERA in 1975 [6]. The only contrary result is the 1972 DISCOS drag-free flight [7] where no proof-mass charging was observed. DISCOS (proof-mass radius = 11 mm, gap = 9 mm, offset = 0.7 mm) should have been able to detect charges as small as 900 mV (3×10^{-12} coul or 2×10^7 electrons). Theoretical calculations using the Creme96 program (<http://crsp3.nrl.navy.mil/creme96>) which can calculate geomagnetic shielding, trapped proton flux (using the AP8 models), and proton flux after internal shielding show that a the 750-km polar orbit of DISCOS should have resulted in a charge of 350 mV (10^{-12} coul or 6×10^6 electrons) with each pass through the SAA, so an unknown mechanism was discharging the DISCOS proof mass.

Cremer96 also shows that for a future EP experiment, a 500-km equatorial orbit would experience no charging from 100 to 200 MeV protons because of geomagnetic shielding and the altitude dependence of the SAA. Thus it is not known at this time what mechanism would charge the proof masses. Nevertheless the experiment must be capable of measuring proof-mass charges and of discharging them. In the case of a space-station orbit, 380 km and 51.4-degree inclination, the calculations show that a shield of 100 gm/cm² of Invar (about 150 kg in a spherical shell 30 cm in radius) would allow no charging in 10⁶ sec (about 12 days) beyond about 2 mV (2×10^{-14} coul or 10⁵ electrons, proof-mass radius = 15 mm, gap = 5 mm).

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