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Separating Out the Radial-Offset Error in a Two-Sphere Equivalence-Principle Experiment in a Satellite

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Abstract: Two concentric spheres free falling in a drag-free satellite could be used to detect a violation of the equivalence principle; but if the gravitation centers of the spheres are not known to high accuracy, a radial offset error could mimic an equivalence principle violation. One solution to this problem is to use local gravity gradients from a spinning satellite to excite a signal at two times roll frequency and to measure the amplitude of this signal. The twice-roll signal provides a measurement of the *x-y* coordinates which is independent of the readout biases. One of the techniques discussed in this paper known as Zero-*z* Excitation does not work nearly as well as a different approach known as DC Cancellation, but a combination of the two known as AC Cancellation has the high precision of DC Cancellation and provides a cross check on the results from the excitation signal.

One obvious design for a high precision equivalence-principle (EP) experiment would be a drag-free satellite [1] with two free-floating concentric spherical proof masses, a solid sphere inside of a spherical shell. Until now this design has not been practical because no way was known to measure the positions of the two proof masses independently, because a violation of the equivalence principle and a radial bias error are not separately observable, and because the response to an equivalence-principle violation only grows as t whereas the largest disturbance, brownian motion from random gas collisions, grows as $t^{3/2}$.

The solution to the measurement problem using a two-color transcollimator is discussed in a separate paper [2]. The observability problem arises because if f_x is the specific force corresponding to a violation of the equivalence principle, and if x_0 is an unknown initial offset in the radial position difference between the two spheres; then in the absence of local gravity gradients in the satellite, f_x and x_0 always show up in the output in the combination $3n^2x_0 + f_x$ where n is the mean orbit rate. In order to detect a violation of the equivalence principle of 10^{-20} g, it would be necessary to know the relative radial offset of the two spheres to between 10^{-13} and 10^{-14} meters. Since they can only be polished to an accuracy of about 10^{-8} meters, the error in the knowledge of their effective gravitation centers could exceed the accuracy requirements of a 10^{-20}-g experiment by 5 or 6 orders of magnitude. That the response to f_x only grows as t not t^2 can be seen from the solution to the Euler-Hill equations, $\ddot{x} - 3n^2x - 2n\dot{y} = f_x$ and $2n\dot{x} + \ddot{y} = 0$ where x and y are relative positions of the two spherical proof masses in the radial and orbit-velocity directions respectively. The x-solution contains only periodic terms, and the y-solution is $y = -2\,(3\,x_0 + f_x/n^2)\,(nt - \sin nt)$.

What is needed is someway to observe the true x-y-difference in the "gravitational centers" of the proof masses without the bias errors of the readout system. One solution is to look at the twice-roll excitation caused by local gravity gradients in the satellite. That the true values of x and y excite such terms can be seen from the differential equations of relative motion. If the satellite spin is normal to the orbit plane with $s2$ and $c2$ being the sine and cosine of twice the satellite roll angle and if the g_{ii} are the gravity-gradient tensor eigenvalues, then under the assumption that the satellite masses have been trimmed to make the gravity-gradient tensor diagonal, the equations of relative motion of the proof masses are

$$\ddot{x} - \left(3n^2 + \frac{g_{11}+g_{22}}{2} + \frac{g_{11}-g_{22}}{2}c2\right)x - \left(\frac{g_{11}-g_{22}}{2}s2\right)y - 2n\dot{y} = f_x, \qquad (1)$$

$$-\left(\frac{g_{11}-g_{22}}{2}s2\right)x + 2n\dot{x} + \ddot{y} - \left(\frac{g_{11}+g_{22}}{2} - \frac{g_{11}-g_{22}}{2}c2\right)y = 0, \qquad (2)$$

where x is the radial direction and y is the coordinate parallel to the orbital velocity. If ω_R is the satellite roll rate, then the twice-roll-rate component in the x-position measurement, s_x, is approximately given by $s_x \approx (g_{11}-g_{22})\,f_x \,/\, 5\,[6n^2 + (g_{11}+g_{22})]\omega_R^2$. In order to have a sensible signal, $g_{11} - g_{22}$ should be as large as possible which requires relatively large masses next to the proof masses. There are two practical choices for the eigenvalues, $[1.01\, g_{11}, -g_{11}, -0.01\, g_{11}]$ and $[-12\, n^2, 6\, n^2, 6\, n^2] \times 0.9983$. There are a number of ways to implement this, but the two most practical are four specially arranged solid Platinum or Osmium spheres for the first choice and a Platinum or Osmium torus with its symmetry axis perpendicular to the spin axis for the second. Figure 1 shows both cases. The dark masses can be interpreted either as a cross section of the torus, or as two of the four spheres. The second pair of spheres would be on the z-axis and are not shown in the figure. The total mass necessary ranges from 20 to 100 kg depending on the size of the proof masses.

The first choice for the eigenvalues is known as Zero-z Excitation, and the coefficient 1.01 allows the value of g_{11} to be as large as $10^{-5}/\mathrm{sec}^2$ without having the system be unstable in any axis. The second possibility makes $(g_{11} + g_{22})/2 = -3\, n^2$ which is the condition for cancellation as can be seen from Equation 1. Since $(g_{11} - g_{22})/2 = -9\, n^2$, there is still an excitation signal at twice roll; and thus this choice is called AC cancellation. The $6\, n^2$ term in the z-axis of AC cancellation implies that unlike the first choice, the system is unstable in the z-axis perpendicular to the experiment plane and must be stabilized with the x-y transcollimator light pressure. The numerical factor 0.9983 is a small correction to the above conditions which is necessary with AC cancellation due to a slight shift in frequency caused by the AC terms.

With a roll period of 100 seconds, $f_x = 10^{-18}\, g$ gives an amplitude for s_x of about 10^{-13} to 10^{-14} meters; and if the roll period could be increased to 1000 seconds without excessive disturbances, the signals would be approximately 100 times greater. Using a two-color transcollimator, 10^{-14} meters can be detected with an averaging time of about 10^4 seconds. The sensitivity quoted above depends on having a long roll period, but short roll periods are desirable to suppress certain of the disturbances. For this reason the technique discussed here is not as good as DC cancellation [3], but the use of the twice-roll signal could provide important confirmation if a relatively large violation of the equivalence principle (10^{-15} to $10^{-19}\, g$) were detected using an experiment based on AC cancellation.

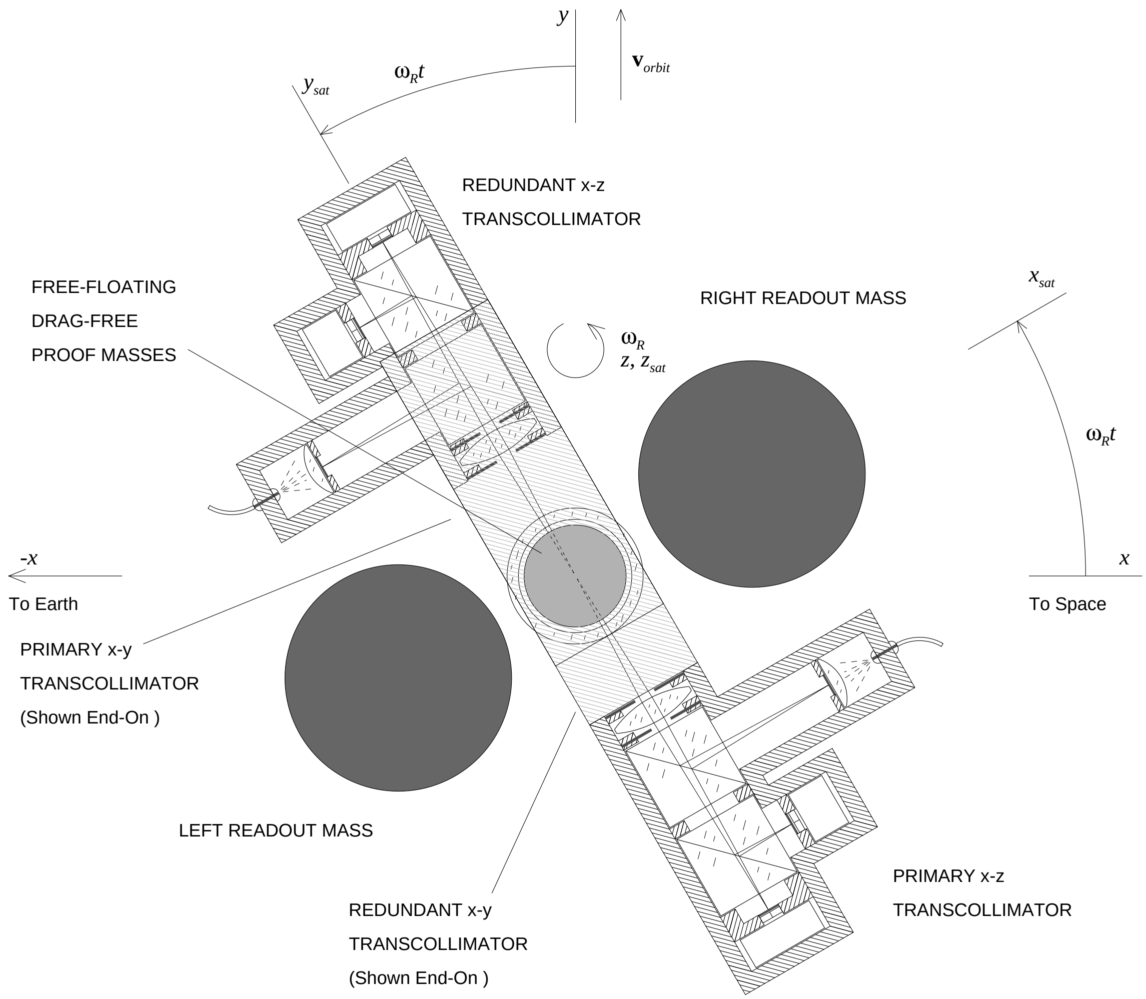

Figure 1. Concentric-Sphere Geometry with Gravity-Gradient Masses

[1] Lange, B., Ph.D. Thesis, Stanford University, Stanford University Department of Aeronautical Engineering Report (SUDAER) No. 194, June 1964. AIAA Journal, **2**, 9, 1950 (1964).

[2] Lange, B., *The Two-Color Transcollimator, a Precision Position Detector for a Satellite Two-Sphere Equivalence-Principle Experiment*, companion paper in this volume.

[3] Lange, B., *Suppressing Radial Non-Observability in a Concentric Two-Sphere Equivalence-Principle Experiment by Gravity-Gradient Cancellation*, companion paper in this volume.