

Suppressing Radial Non-Observability in a Concentric Two-Sphere Equivalence-Principle Experiment by Gravity-Gradient Cancellation

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Abstract: A design for an equivalence-principle experiment in a drag-free satellite using two free-falling concentric spheres is discussed. In addition to the problem of measuring the positions of the spheres, this approach suffers from the difficulty that a small radius error can not be distinguished from a violation of the equivalence principle (EP) and the response to an EP violation only grows as t not t^2 . Both of these problems can be solved by adding heavy masses on each side of the cavity to cancel the earth's gravity gradient and the differential centrifugal force. The resulting system could detect a violation of the equivalence principle between 10^{-20} and $10^{-24} g$. Although the experiment is located in a satellite orbiting the earth, it is equivalent to a very tall (0.1 to 10 AU) drop tower with drop times between 10^5 and 10^6 seconds and an effective value of g equal to about 3.4 meters/sec² which is 3/7 of the value of g at the orbital altitude.

One obvious design for a high precision equivalence-principle experiment would be a drag-free satellite [1] with two free-floating concentric spherical proof masses, a solid sphere inside of a spherical shell. This approach has not been considered until now because of the three problems discussed above and in [2]. Of these, a solution to the measurement problem is given in [3].

The radial nonobservability occurs because the combination $3n^2x_0 + f_x$ always occurs together in the y -response to an EP violation. Here f_x is a small specific force term which is used to model the EP violation, x is the coordinate in the radial direction, y is parallel to the orbit velocity, and n is the mean orbit rate. The key to these two problems, no- t^2 response and radial nonobservability, is to realize that they do not arise in the orbit dynamics because of the earth's gravitational field but because of its gradient. One possible solution is to modify the gravitational gradient at the center of the proof masses to suppress these difficulties. This is possible because while there is a very great difference between the gravitational field of the earth and the self gravity of laboratory masses, their gravity gradients are of comparable sizes. If the spin of the satellite is normal to the orbit plane, two solid spheres fixed in the satellite structure on each side of the proof masses on a line parallel to the spin axis can cancel the combined earth gravity gradient and the differential centrifugal force in a locally-level reference frame. Figure 1 of [2] can be reinterpreted to show this case too, if x_{sat} of that figure is assumed to be the z -axis, i.e. the satellite spin axis. In the ideal case if the combined gravity gradient of the two spheres is chosen to be $-3n^2$ perpendicular to the spin axis, the relative equations of the proof masses become

$$\ddot{x} - 2n\dot{y} = f_x$$

and

$$2n\dot{x} + \ddot{y} + 3n^2y = 0$$

instead of the usual Euler-Hill equations (see [2]). When this is done, both the non-observability and the t -only dependence are destroyed, i.e. the solution for x now contains a term $3f_x t^2 / 14$ and there is no connection between x_0 and f_x . This approach is known as

DC Cancellation because the twice-roll terms which would normally appear in the equations (see [2], Equations 1 and 2) are eliminated by the two-mass configuration along the satellite spin axis. In the non-ideal case where the cancellation is not perfect, if the g_{ii} are the eigenvalues of the local gravity gradient tensor due to the satellite masses and if $k^2 = 3n^2 + (g_{11} + g_{22})/2$ is the error in the cancellation, then the combination $f_x + k^2 x_0$ appears in the solution; and the nonobservability is suppressed by the ratio $k^2/3n^2$. Prior drag-free satellite experience indicates that cancellation to about one part in 10^4 to 10^6 is possible [4], and this level of precision would mean that the t^2 response holds between 10^5 and 10^6 seconds. Choosing $g_{11} + g_{22} = -6n^2$ makes the z -axis unstable with a time constant of about 450 seconds, so that z must be controlled to zero using the transcollimator light pressure.

Using this approach the accuracy of the experiment is limited by the disturbances and not by the transcollimator readout accuracy. A Kalman filter covariance calculation using the disturbance from random gas collisions and the transcollimator output noise shows that for measurement times between 10^6 and 10^8 seconds, an equivalence principle violation between 10^{-20} and 10^{-24} g could be detected depending on the quality of the vacuum. If a low-temperature system is compared with a low-temperature system, the approach discussed here is about 10^4 times more accurate than an experiment based on a cylinder constrained to move along a single axis [5]; and even a room temperature system is about 10 times more accurate than a low-temperature cylindrical experiment provided that a vacuum of 10^{-9} Torr can be maintained. This paper gives only a brief overview of this approach, and a much more detailed description of this method is being prepared for publication including a discussion of the disturbances and the cancellation accuracy [6].

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